E L B S

An Introduction to Electrical Circuit Theory

G. Williams

LOW-PRICED EDITION

An Introduction to
Electrical Circuit Theory

An Introduction to

Electrical Circuit Theory

G. Williams

Lecturer in Electrical Engineering
School of Applied Sciences
University of Sussex

Macmillan Education

First edition 1973
Reprinted 1975 (with corrections), 1977

ELBS edition first published 1977

Published by
THE MACMILLAN PRESS LTD
London and Basingstoke
Associated companies in Delhi Dublin
Hong Kong Johannesburg Lagos Melbourne
New York Singapore and Tokyo

ISBN 978-0-333-23385-6 ISBN 978-1-349-03637-0 (eBook)
DOI 10.1007/978-1-349-03637-0

Contents

Preface ix

1 BASIC DEFINITIONS AND CIRCUIT LAWS 1

1.1 Units and dimensions 1
1.1.1 Electrical quantities 1
1.1.2 Dimensional analysis 3
1.2 The circuit element of resistance 4
1.2.1 Ohm's law 5
1.2.2 Non-linear resistors 7
1.2.3 Power dissipated by resistors 7
1.3 Kirchhoff's laws 8
1.3.1 Sign conventions 8
1.3.2 Ideal circuit elements 9
1.3.3 Current law 10
1.3.4 Voltage law 11
1.3.5 Applications of Kirchhoff's laws 12
Problems 16

2 METHODS OF ANALYSIS AND NETWORK THEOREMS 18

2.1 Terminology and network topology 18
2.2 The basic method of network analysis 21
2.2.1 Worked example 22
2.3 Mesh analysis 23
2.3.1 Mesh currents 23
2.3.2 Method of mesh analysis 24
2.3.3 Worked example 25
2.3.4 General solution 25
2.4 Nodal analysis 26
2.4.1 Method of nodal analysis 27
2.4.2 Worked example 27
2.4.3 General solution 28
2.4.4 Mesh or nodal analysis? 29

2.5 Duality 29
2.5.1 Dual networks 29
2.5.2 Sign conventions 31
2.5.3 Worked example 32
2.6 Sources 32
2.6.1 Representation of physical sources 33
2.6.2 Source equivalence 35
2.6.3 Compound sources 35
2.6.4 Resistanceless sources 36
2.6.5 Maximum power transfer theorem 39
2.7 Network theorems 40
2.7.1 Linearity 40
2.7.2 Superposition theorem 41
2.7.3 Star–delta transformation 43
2.7.4 Thévenin's theorem 47
2.7.5 Norton's theorem 49
2.7.6 Reciprocity theorem 51
2.8 Equivalent T and Π circuits 52
2.8.1 Two-port parameters 53
2.8.2 Equivalent T circuits 54
2.8.3 Equivalent Π circuits 55
2.8.4 Π–T equivalence 56
Problems 56

3 THE CIRCUIT ELEMENTS OF CAPACITANCE AND INDUCTANCE

3.1 Capacitance 61
3.1.1 Definition 61
3.1.2 Volt-ampere equations 62
3.1.3 Energy stored 63
3.1.4 Capacitances in parallel 64
3.1.5 Capacitances in series 65
3.2 Inductance 66
3.2.1 Definition of self-inductance 66
3.2.2 Volt–ampere equations 67
3.2.3 Energy stored 68
3.2.4 Inductances in series 68
3.2.5 Inductances in parallel 69
3.3 Duality 70
3.4 Mutual inductance 70
3.4.1 Definition of mutual inductance 70
3.4.2 Volt–ampere equations 71
3.4.3 Equivalent voltage generators 72
3.4.4 Coefficient of coupling 73
3.4.5 Magnetic energy stored by mutual inductance 73
Problems 75

4 TRANSIENT RESPONSE OF FIRST- AND SECOND-ORDER NETWORKS 77

4.1 Differential equations of first-order networks 78
4.1.1 Initial conditions 78
4.1.2 Final conditions 80
4.1.3 Application of the basic analytical method 81
4.1.4 Mesh analysis 82
4.1.5 Nodal analysis 82
4.1.6 Application of network theorems 83
4.2 Natural response of first-order networks 83
4.2.1 *RC* circuit 83
4.2.2 *RL* circuit 85
4.3 Some steady-state responses of first-order networks 86
4.3.1 Series *RL* circuit excited by a step function 86
4.3.2 Series *RC* circuit excited by a step function 87
4.3.3 Series *RL* circuit excited by a sinusoidal function 87
4.4 Total response of first-order networks 89
4.4.1 Step response of *RL* circuits 89
4.4.2 Repetitive step response of *RL* circuit 91
4.4.3 Step response of *RC* circuit 92
4.4.4 Repetitive step response of *RC* circuit 93
4.5 Differential equations of second-order networks 95
4.6 Natural response of second-order networks 96
4.6.1 Overdamped case 98
4.6.2 Critically damped case 99
4.6.3 Underdamped case 100
4.6.4 Zero damping case 101
4.6.5 Definitions 101
4.7 Step response of second-order networks 102
4.8 *Q*-factor 104
Problems 104

5 STEADY-STATE SINUSOIDAL RESPONSE OF NETWORKS 110

5.1 Response of series *RLC* circuit 110
5.2 Frequency response 113
5.2.1 Amplitude 113
5.2.2 Phase 113
5.3 *Q*-factor, bandwidth and resonance 114
5.4 A.C. quantities 118
5.4.1 Instantaneous, mean and r.m.s. values 119
5.4.2 Real and reactive power 122
5.5 Representation of sinusoids 127
5.5.1 Rotating vectors 127
5.5.2 Complex number representation of sinusoids 132

5.6 Frequency-domain analysis 136
5.6.1 Volt–ampere equations in the frequency domain 136
5.6.2 Kirchhoff's laws in the frequency domain 140
5.6.3 Mesh and nodal analysis 141
5.6.4 Impedance and admittance 144
5.7 Worked example 145
5.7.1 By phasor diagram 145
5.7.2 By nodal analysis 147
5.7.3 By Thévenin substitution 149
5.8 Power in the frequency domain 150
5.8.1 Real, reactive and apparent power 150
5.8.2 Power triangle 151
5.8.3 Worked example 154
5.8.4 Maximum power transfer theorem 156
Problems 158

6 NETWORK ANALYSIS TECHNIQUES 164

6.1 Two-port networks 164
6.1.1 Open-circuit impedance parameters 165
6.1.2 Short-circuit admittance parameters 166
6.1.3 Hybrid parameters 167
6.1.4 Inverse-hybrid parameters 167
6.1.5 Transmission parameters 168
6.1.6 Interrelation of parameters 169
6.1.7 Equivalent T and Π networks 171
6.2 Ideal transformer 173
6.2.1 Volt–ampere equations 173
6.2.2 Impedance transformation 175
6.2.3 Voltage and magnetic flux waveforms 176
6.2.4 Phasor diagrams 177
6.2.5 Worked example 179
6.3 Bridge networks 182
6.3.1 Wheatstone bridge 183
6.3.2 Maxwell bridge 183
6.4 Polyphase circuits 184
6.4.1 Balanced three-phase voltages 184
6.4.2 Balanced three-phase currents 188
6.4.3 Three-phase power 191
6.5 Non-linear circuits–load line and linear approximation 192
6.5.1 Non-linear elements 193
6.5.2 Analytical techniques 194
Problems 198

Solutions to Problems 200

Index 205

Preface

Many textbooks owe their origins to undergraduate lecture courses; this book had its beginnings in a lecture course in engineering science given by the author at the University of Sussex. When the course was begun several years ago the familiar problem of not being able to recommend a single, inexpensive, book to the students taking the course was encountered. The nature of the course structure at Sussex with its major and minor subjects made the problem more difficult because the students attending the course included not only electrical and electronic engineers but undergraduates from many other scientific disciplines too. Educational experiments in the presentation of course material being conducted at the time also meant that printed lecture notes were prepared for the course and it is with these that this book had its beginnings.

While the course contained much circuit theory other topics were included which do not appear here and conversely this book contains several topics not covered in the course. The additional material has been included so that the book may be considered to be an introduction to the subject of circuit theory viewed as a separate discipline and not viewed as a service subject to other disciplines.

The philosophy governing the presentation of the material is that all the circuit laws, methods of analysis and circuit theorems are developed using the simplest possible circuits containing only resistances and d.c. sources. Thus the discussion is not clouded by the examination of the more complicated circuit elements and sources which introduce time variations; the intention is for the student to master the analytical techniques before he goes on to apply them to the more complicated circuits. Application of the techniques to frequency domain circuits is then a logical step which allows the student to concentrate on the frequency domain concepts.

The background knowledge of readers is assumed to be school mathematics and physics but to include no circuit theory. It is also assumed that students will be covering such topics as complex numbers, second-order differential equations and linear algebra simultaneously with their studies of circuit theory and such mathematical topics are not discussed in detail here.

I would like to acknowledge the advice, co-operation and support I have received from my colleagues at the University of Sussex. It is also no cliché to say that this book could not have been created without the active assistance, tolerant

understanding and unflagging encouragement of my wife.

Many of the problems in this book were originally devised by me for Preliminary Year examinations of the University of Sussex and I am grateful to the University of Sussex for permitting me to use them.

G. W.

1

Basic Definitions and Circuit Laws

1.1 Units and dimensions

It may be shown that all *quantities* required in mechanics may be expressed in terms of three basic physical quantities. Although many sets of three quantities may be used as the basic one, the most natural, and the one most universally used is that of mass, length and time. In the SI (Système Internationale d'Unités) system of units the fundamental *units* associated with the basic physical quantities are the metre, kilogram and second which are defined respectively in terms of the wavelength of the radiation of the orange line of krypton 86; the mass of a particular block of platinum located at Sèvres in France; and the period of oscillation of the caesium atom. All other mechanical quantities may be expressed in terms of the three basic quantities raised to various powers, which may be positive or negative, and which are termed the *dimensions* of the derived quantity. For example, force is defined in terms of the acceleration imparted to a given mass and hence the dimensional equations are

$$
\begin{aligned}
[\text{force}] &= [\text{mass}]\,[\text{acceleration}] \\
&= [M]\left[\frac{L}{T^2}\right] \\
&= [MLT^{-2}]
\end{aligned}
$$

and the dimensions of force may be written as

$$[1, 1, -2]$$

The study of electrical phenomena, illumination and systems involving heat flow require additional basic quantities and fundamental units. The additional basic quantities of the SI are electric current, luminous intensity and temperature and the additional fundamental units are the ampere, the candela and the Kelvin.

1.1.1 Electrical quantities

The fourth fundamental unit of the SI is the *ampere*, symbol A, which is now defined in terms of its magnetic effect as 'the intensity of a constant current which, when maintained in two parallel straight conductors of infinite length and negligible cross-section placed at a distance of one metre apart in vacuo, produces between them a force of 2×10^{-7} newton per metre length'.

The unit of current may be defined in other ways with reference to other physical effects associated with it. An earlier definition was concerned with electrolysis, or the chemical effect of current, and referred to the amount of silver deposited by passing a current through a solution of silver nitrate. Other possibilities include the heating effect associated with current flowing in conductors; a definition based upon energy; and the force exerted by charged particles at rest. All other electrical quantities may be defined with reference to their physical effects in terms of the basic quantities of mass, length, time and current and again the powers to which the basic quantities are raised to obtain the desired quantity are termed the dimensions of the quantity. Some of the definitions of electrical quantities follow.

Electric charge, symbol Q. The ultimate unit of charge is the electron which has a mass of approximately 9.1×10^{-31} kg and possesses a negative charge. An electric current consists of a flow of electric charge through a conducting region and hence the integral of current with respect to time is charge. The unit of charge is the *coulomb*, which is defined as the charge transferred by a current of one ampere in one second. Conversely, the ampere may be defined as the flow of one coulomb of charge through a cross-section in one second. Hence

$$i = \frac{\mathrm{d}q}{\mathrm{d}t}$$

and the dimensions of charge are

$$[Q] = [TI]$$

Potential difference, symbol V. The unit of potential difference is the *volt,* symbol V, and it is defined in terms of energy. The flow of an electric charge may have associated with it a transfer of energy and therefore when a charge is moved from one point to another the electric potential difference existing between those points may be defined in terms of the energy required to move the charge. The volt is that electric potential difference existing between two points if a charge of one coulomb receives or delivers one joule of energy in moving between them.

The dimensions of energy are

$$[ML^2T^{-2}]$$

and the dimensions of potential difference are therefore

$$[V] = \frac{[W]}{[Q]}$$

$$= \frac{[ML^2T^{-2}]}{[TI]}$$

Hence

$$[V] = [ML^2T^{-3}I^{-1}]$$

Electric power, symbol P. It follows from the definition of potential difference that if charge is continuously transported from one point to another, through a potential difference, energy is continuously transferred. As power is the rate of doing work, the power associated with the transportation of charge is given by the product of voltage and current. Instantaneously

$$p = \frac{dw}{dt} \frac{\text{joules}}{\text{second}}$$

but

$$v = \frac{dw}{dq} \frac{\text{joules}}{\text{coulomb}}$$

and

$$i = \frac{dq}{dt} \frac{\text{coulombs}}{\text{second}}$$

hence

$$p = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = vi \text{ watts}$$

The total energy is therefore

$$w = \int p \, dt = \int vi \, dt \text{ joules}$$

1.1.2 Dimensional analysis

The technique of dimensional analysis is a powerful tool for checking the results of analysis and calculation. It may also be used to deduce the form of an equation describing a physical phenomenon. The technique is based upon the necessity for every term in an analytical equation to have the same dimensions.

As an example, consider an electric circuit consisting of a resistance and an inductance in series connected to a time varying electric potential. The application of Kirchhoff's voltage law (see later) results in the equation

$$v = Ri + L\frac{di}{dt}$$

The principle of dimensional homogeneity demands that the two terms on the right-hand side of the equation must have the dimensions of voltage. The dimensions of resistance are

$$[ML^2T^{-3}I^{-2}]$$

and hence the dimensions of (Ri) are

$$\begin{aligned}[Ri] &= [ML^2T^{-3}I^{-2}]\,[I] \\ &= [ML^2T^{-3}I^{-1}]\end{aligned}$$

which are the dimensions of electric potential. Similarly the dimensions of $[L(di/dt)]$ are

$$\begin{aligned}\left[L\frac{di}{dt}\right] &= [ML^2T^{-2}I^{-2}]\left[\frac{I}{T}\right] \\ &= [ML^2T^{-3}I^{-1}]\end{aligned}$$

which again are the dimensions of electric potential.